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Problem Solutions

View detailed solutions to each problem. Click on "Show Solution" to see the full explanation.



(1)
$$d^2y/dx^2 = (\rho g/S) x$$

(2)
$$d^2y/dx^2 = (\rho g/S) y$$

(3) $d^2y/dx^2 = \sqrt{(\rho g/S)}$

(4) $dy/dx = \sqrt{(\rho g/S) x}$

Hide Solution

Solution:

Here is a clean derivation of the differential equation for the shape of the meniscus. We shall use

1. Young–Laplace: the pressure jump across a curved interface of surface-tension ${\cal S}$ is

 $\Delta p \;=\; S\,\kappa$

where κ is the curvature of the free surface.

2. Hydrostatics in the liquid: measured from the level y=0 where $p=p_{
m atm}$, the liquid pressure at height y is

$$p_{
m liq}(y) \;=\; p_{
m atm} +
ho\, g\, y\,.$$

3. Small-slope approximation: since $\theta \ll 1$,

$$\kappa \;=\; rac{y''}{ig(1+(y')^2ig)^{3/2}} \;pprox\; y''\,, \quad y'= an hetapprox hetapprox heta\,.$$

Step 1: Pressure jump = hydrostatic pressure

$$\Delta p \;=\; p_{\mathrm{liq}} - p_{\mathrm{atm}} \;=\;
ho\, g\, y(x).$$

Step 2: Also by Young–Laplace,

$$\Delta p \;=\; S\,\kappa\;pprox\; S\,y''(x).$$

Equate them:

$$egin{array}{lll} \displaystyle rac{d^2y}{dx^2} &= \displaystyle rac{
ho\,g}{S}\,y\,. \end{array}$$

 $S\,y^{\prime\prime}(x)\ =\
ho\,g\,y(x),$

or

Thus choice (2) is the correct differential equation.

If one wanted the explicit shape one writes

$$y''-\lambda^2\,y=0,\quad\lambda\equiv\sqrt{rac{
ho g}{S}},$$

SO

$$y(x) = A \, e^{\lambda x} + B \, e^{-\lambda x},$$

and the two constants A,B are fixed by the two contact-angle (slope) conditions at the walls.

A microscope has an objective of focal length 2 cm, eyepiece of focal length 4 cm and the tube length of 40 cm. If the distance of distinct vision of eye is 25 cm, the magnification in the microscope is

(1) 100				
(2) 123				
450				
(3) 150				
(4) 250				
THide Solution				
Solution:				
Here is the step-by-step solution using standard microscope formulas				
Step 1: Magnification by the objective				
The lateral magnification produced by the objective is				
tube length l 40 cm				
$M_o = \frac{\text{tube length } t}{\text{objective focal length } f} = \frac{40 \text{ cm}}{2 \text{ cm}} = 20.$				
$J_0 = 2 \text{ cm}$				
Oten O. Annular meanification but the evening of				
Step 2. Angular magnification by the eyepiece				
relaxed-eve condition), the angular magnification is				

$$M_e \;=\; rac{D}{f_e} \;=\; rac{25 \; {
m cm}}{4 \; {
m cm}} \;=\; 6.25 \,,$$

where D = 25 cm is the near-point (distinct-vision) distance.

Step 3: Total magnification The overall magnification of the microscope is the product

 $M = M_o imes M_e = 20 imes 6.25 = 125$.

Answer: 125 (option 2)

3 Problem 3

An electron (mass 9×10^{-31} kg and charge 1.6×10^{-19} C) moving with speed c/100 (c = speed of light) is injected into a magnetic field B of magnitude 9×10^{-4} T perpendicular to its direction of motion. We wish to apply a uniform electric field E together with the magnetic field so that the electron does not deflect from its path. Then (speed of light c = 3×10^8 ms⁻¹)

(1) E is perpendicular to B and its magnitude is 27 × 10⁴ V m⁻¹

(2) E is perpendicular to B and its magnitude is 27×10^2 V m⁻¹

(3) E is parallel to B and its magnitude is 27×10^2 V m⁻¹

(4) E is parallel to B and its magnitude is 27×10^4 V m⁻¹

Hide Solution

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Solution:

Solution:

Step 1: Compute the speed of the electron. The electron moves at

$$v = rac{c}{100} = rac{3 imes 10^8 \ {
m m/s}}{100} = 3.0 imes 10^6 \ {
m m/s} \,.$$

Step 2: Write the condition for zero net force. The total force on a charge q in electric and magnetic fields is

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} imes \mathbf{B}) \,.$$

For no deflection, ${f F}=0$, so

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0 \implies \mathbf{E} = -\mathbf{v} \times \mathbf{B}.$$

Step 3: Determine the magnitude of \mathbf{E} . Since the velocity \mathbf{v} is perpendicular to \mathbf{B} , the magnitude is

$$E = |\mathbf{v} \times \mathbf{B}| = v B \sin 90^\circ = v B.$$

Substituting $v = 3.0 imes 10^6$ m/s and $B = 9.0 imes 10^{-4}$ T gives

$$E = (3.0 imes 10^6)(9.0 imes 10^{-4}) = 2.7 imes 10^3 \ {
m V/m} = 27 imes 10^2 \ {
m V/m}$$
 .

Step 4: State the orientation of \mathbf{E} .

From $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$, the electric field is perpendicular to both \mathbf{v} and \mathbf{B} . In particular, it is perpendicular to \mathbf{B} .

Therefore the correct choice is:

(2) $E\perp B~~{
m and}~~E=27 imes 10^2~{
m V/m}$.

There are two inclined surfaces of equal length (L) and same angle of inclination 45° with the horizontal. One of them is rough and the other is perfectly smooth. A given body takes 2 times as much time to slide down on rough surface than on the smooth surface. The coefficient of kinetic friction (μ_k) between the object and the rough surface is close to

(1)	0.25			
(2)	0.40			
(3)	0.5			
(4)	0.75			
T Hide Solution				
Solution: Here is a detailed solution. We denote • the length of each plane by L , • the inclination angle by $\theta = 45^{\circ}$, • the acceleration due to gravity by g ,				
• the • the • the Step	e time to descend the smooth plane by t_s , e time to descend the rough plane by t_r .			
On the smooth plane there is no friction, so the component of gravity along the plane				

$$a_s \;=\; g \sin heta \;=\; g \sin 45^\circ \;=\; rac{g}{\sqrt{2}} \,.$$

Starting from rest and travelling distance L under constant acceleration a_s ,

$$L \;=\; rac{1}{2}\,a_s\,t_s^2 \;\; \Longrightarrow \;\; t_s \;=\; \sqrt{rac{2L}{a_s}} \;=\; \sqrt{rac{2L}{g/\sqrt{2}}} \;=\; \sqrt{rac{2\sqrt{2}\,L}{g}} \,.$$

Step 2. Motion on the rough plane

On the rough plane the frictional force is $\mu_k mg \cos \theta$ up the slope, so the net acceleration down the plane is

$$a_r = g \sin \theta - \mu_k g \cos \theta = rac{g}{\sqrt{2}} (1 - \mu_k).$$

Again starting from rest and travelling distance L,

$$L = rac{1}{2} a_r t_r^2 \implies t_r^2 = rac{2L}{a_r} = rac{2L}{rac{g}{\sqrt{2}}(1-\mu_k)} = rac{2\sqrt{2} L}{g(1-\mu_k)} \,,$$

SO

$$t_{r} \ = \ \sqrt{rac{2\sqrt{2}\,L}{g\,(1-\mu_{k})}} \,.$$

Step 3. Using the condition $t_r=2\,t_s$

We are told the body takes twice as long on the rough plane:

$$t_r = 2 t_s \implies t_r^2 = 4 t_s^2.$$

Substitute our expressions for t_r^2 and t_s^2 :

$$rac{2\sqrt{2}\,L}{g\,(1-\mu_k)} \;=\; 4\; imes\;rac{2\sqrt{2}\,L}{g}$$



Here is a step-by-step solution using the work–energy principle:

1. **List the given data.**

Car A: initial kinetic energy $K_A = 100$ J, stopping distance $d_A = 1000$ m. Car B: initial kinetic energy $K_B = 225$ J, stopping distance $d_B = 1500$ m.

Apply the work–energy theorem.
 The work done by the braking force brings the car to rest, so

$$F d = \Delta K = -K_{ ext{initial}},$$

 $F = \frac{K}{d}$.

where we take magnitudes. Thus for each car

3. **Compute F_A and F_B .**

$$F_A = rac{K_A}{d_A} = rac{100}{1000} = 0.10 \ \mathrm{N},$$

$$F_B = rac{K_B}{d_B} = rac{225}{1500} = 0.15 \ {
m N}.$$

4. **Form the ratio $\frac{F_A}{F_B}$.**

$$rac{F_A}{F_B} = rac{0.10}{0.15} = rac{10}{15} = rac{2}{3}.$$

5. **Answer.** The ratio $F_A: F_B$ is 2:3, i.e.



Hide Solution

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Solution:

Here is one convenient way to see that the battery-current is 0.5 A. We shall reduce the purely-resistive part of the network between C and D to a single resistance R_1 , combine all the resistances in series with the battery into R_2 , and then write a single node-equation for the voltage $V \equiv V_C - V_D$.

1) Label the important nodes.

Call the top-right junction "C" and the lower-right junction "D" (these are the two terminals of the battery branch). All the rest of the network is purely resistive, connecting C to D through two parallel routes.

2) Simplify the resistive network between C and D (ignoring the battery branch).

(a) Between B and C there are two resistors in parallel, $2.5~\Omega$ and $1.5~\Omega.$ Their equivalent is

$$R_{BC} \;=\; ig(2.5 \parallel 1.5ig) \;=\; rac{2.5 \cdot 1.5}{2.5 + 1.5} \;=\; 0.9375 \; \Omega.$$

(b) Between B and D there are two routes:

- a direct $6 \ \Omega$ branch,

- a longer branch B
ightarrow A
ightarrow E
ightarrow D of $5+3+1.5=9.5~\Omega.$

These two are in parallel, so

$$R_{BD} \;=\; ig(6 \parallel 9.5 ig) \;=\; rac{6 imes 9.5}{6 + 9.5} \;=\; rac{57}{15.5} \;pprox\; 3.677 \,\Omega.$$

(c) These two equivalents, R_{BC} and R_{BD} , sit *in series* along the only purely-resistive route from C to D. Hence the total purely-resistive C–D resistance is

$$R_1 = R_{BC} + R_{BD} = 0.9375 + 3.677 = 4.6145 \, \Omega \quad (pprox 4.615 \, \Omega).$$

3) Identify the battery branch and its series resistance. On the right-hand side C–D is also connected by

$$5.5 \,\Omega \ + \ rac{1}{3} \,\Omega$$

in series *with* the $5\,\text{V}$ source. Call the sum of those two resistors

$$R_2 = 5.5 + \frac{1}{3} = 5.8333 \Omega.$$

Thus the network between C and D consists of

– one branch purely resistive $R_1=4.6145~\Omega$,

– another branch $R_2 = 5.8333 \ \Omega$ in series with a 5 V source E.

4) Write KCL at the "supernode" C–D. Let

$$V = V_C - V_D.$$

Define currents positive from C toward D:

- Through the pure-resistor branch:

$$I_1 = rac{V}{R_1}.$$

- Through the battery branch (resistors + source; current from C to D):

$$I_2 = rac{V-E}{R_2} \quad ext{because going from C to D you first drop } V_C - V_D = V,$$

and in that drop is +E across the source and I_2R_2 across the resistors.

KCL at C (no other injections) is

$$I_1 + I_2 = 0.$$

Hence

$$rac{V}{R_1} \ + \ rac{V-5}{R_2} \ = \ 0 \quad \Longrightarrow \quad rac{V}{4.6145} + rac{V-5}{5.8333} = 0.$$

Multiply through by (4.6145)(5.8333) and solve for V:

 $5.8333 \, V \ + \ 4.6145 \, (V-5) = 0 \implies (5.8333 + 4.6145) \, V = 4.6145 imes 5 \implies V = 0$

**5) Find the battery-branch current.*

$$I_2 = \frac{V-5}{R_2} = \frac{2.209-5}{5.8333} = \frac{-2.791}{5.8333} = -0.4786 \text{ A}.$$

The minus sign simply means that our assumed " $C \rightarrow D$ " direction is opposite to the true direction of discharge. In magnitude the battery is delivering

$$|I_2|pprox 0.48~{
m A}~pprox 0.5~{
m A}.$$

-	-	-

Answer:

The current through the 5 V battery is approximately **0.5 A**.

A bob of heavy mass m is suspended by a light string of length I. The bob is given a horizontal velocity v_0 as shown in figure. If the string gets slack at some point P making an angle θ from the horizontal, the ratio of the speed v of the bob at point P to its initial speed v_0 is:



Here is a complete step-by-step solution, using ... for inline math and

• • •

for display math.

1. Geometry and coordinates Let the pivot be at the origin O = (0, 0). Initially the bob is at the "bottom" point

B = (0, -l)

and is given speed v_0 horizontally. At the instant the string goes slack the bob is at point

$$P = (l\cos\theta, \ l\sin\theta)$$

where heta is measured up from the horizontal to the string.

2. Conservation of mechanical energy Take gravitational potential zero at B. Then the vertical rise from B to P is

$$\Delta h = y_P - y_B = l \sin \theta - (-l) = l (1 + \sin \theta).$$

Energy conservation from B to P gives

$$rac{1}{2}\,m\,v_0^2 \ = \ rac{1}{2}\,m\,v^2 \ + \ m\,g\,\Delta h \ = \ rac{1}{2}mv^2 \ + \ mg\,l\,(1+\sin heta)\,.$$

Hence

$$v^2 = v_0^2 - 2 g l (1 + \sin \theta).$$
 (1)

3. "Slack" (zero-tension) condition In polar form the radial equation is

$$\sum F_{
m rad} \;=\; m \, a_{
m rad} \;=\; -m \, rac{v^2}{l} \,,$$

where outward along the string is positive. The two radial forces are tension T (inward, i.e. -T) and the radial component of gravity. Projecting $\vec{F}_g = -mg\hat{j}$ onto

the outward unit-vector

 $\hat{e}_r = (\cos heta, \sin heta)$ gives

$$F_{g,r} = \,ec{F}_g \cdot \hat{e}_r = -mg \,(0 \cdot \cos heta + 1 \cdot \sin heta) = -\, mg \,\sin heta \quad ext{(inward)}.$$

Thus the radial balance is

$$-T - mg\sin heta = -mrac{v^2}{l} \implies T = mrac{v^2}{l} - mg\sin heta.$$

The string goes slack when T=0, so

$$m \frac{v^2}{l} = m g \sin \theta \implies v^2 = g l \sin \theta.$$
 (2)

4. Combine (1) and (2) From (2) substitute $v^2 = g \, l \, \sin heta$ into (1):

$$g \, l \, \sin \theta \; = \; v_0^2 \; - \; 2 \, g \, l \, (1 + \sin \theta) \implies v_0^2 = g \, l \, (\sin \theta + 2 + 2 \sin \theta) = g \, l \, (2 + 3 \sin \theta)$$

Therefore

$$rac{v^2}{v_0^2} = rac{g \, l \, \sin heta}{g \, l \, (2 + 3 \sin heta)} = rac{\sin heta}{2 + 3 \sin heta} \quad \Longrightarrow \quad rac{v}{v_0} = \sqrt{rac{\sin heta}{2 + 3 \sin heta}}.$$

Answer.

$$rac{v}{v_0} = \sqrt{rac{\sin heta}{2+3\sin heta}} \; ,$$

which is choice (4).



Solution:

Here is a complete, step-by-step analysis. We label the two left-hand gates as f_1 (the top one) and f_2 (the bottom one), and the right-hand gate as an OR.

1. Identify the left-hand gates:

– The top-left symbol is an OR-shape with a bubble on its output. That is a NOR gate, so

$$f_1 = A + B.$$

– The bottom-left symbol is an AND-shape with a bubble on its output. That is a NAND gate, so

$$f_2 = \overline{AB}$$

2. The two outputs f_1 and f_2 then feed into a plain OR gate on the right, giving

$$Y = f_1 + f_2 = \overline{A + B} + \overline{AB}$$

3. Simplify algebraically:

$$\overline{A+B} = \overline{A}\overline{B}, \qquad \overline{AB} = \overline{A} + \overline{B},$$

SO

$$Y = \bar{A}\,\bar{B} + (\bar{A} + \bar{B}) = \underbrace{(\bar{A}\,\bar{B} + \bar{A})}_{=\bar{A}} + \bar{B} = \bar{A} + \bar{B} = \overline{AB}.$$

Thus

$$Y = A B$$

which is exactly the NAND function of A and B.

4. We can also see by truth table that the network outputs 1 for all inputs except A = 1, B = 1—again matching NAND.

Therefore the composite circuit implements the NAND function. Answer:

(2) NAND gate.

9 Problem 9

The electric field in a plane electromagnetic wave is given by $E_z = 60 \cos(5x + 1.5 \times 10^9 \text{ t}) \text{ V/m}$. Then expression for the corresponding magnetic field is (here, subscripts denote the direction of the field):



Hide Solution

Solution:

Here is a detailed step-by-step derivation. We shall use Maxwell's equation

$$abla imes {f E} \;=\; - \; rac{\partial {f B}}{\partial t}$$

to find the magnetic field corresponding to

$${E_z}(x,t) = 60 \cos ig(5x + 1.5 imes 10^9 \, t ig) \, {
m V/m} \, .$$

1. Identify the wave parameters.

From the argument of the cosine,

$$k = 5 \; \mathrm{m}^{-1}, \qquad \omega = 1.5 imes 10^9 \; \mathrm{s}^{-1}.$$

The phase $\phi = 5x + 1.5 \times 10^9 t$ has the form $kx + \omega t$, which describes a wave propagating in the -x-direction.

2. Note the field geometry.

E points along *z*, the wave propagates along -x, so **B** must point along *y* so that

 ${f S}=rac{1}{\mu_0}\,{f E} imes{f B}$

goes along -x.

3. Compute the curl of **E**. Only E_z is nonzero and depends on x, so in component form

$$(
abla imes \mathbf{E})_y = \partial_z E_x - \partial_x E_z = 0 - rac{\partial}{\partial x} \Big[60\cos(5x + \omega t) \Big] = -60 \cdot 5 \Big[-\sin(5x + \omega t) \Big]$$

4. Insert into Maxwell's equation.

$$(
abla imes {f E})_y = -rac{\partial B_y}{\partial t} \implies -300 \sin(5x+\omega t) = -rac{\partial B_y}{\partial t} \implies rac{\partial B_y}{\partial t} = 300 \sin(5x+\omega t)$$

5. Integrate with respect to t.

$$B_y(x,t) = \int 300 \sin(5x+\omega t) \, dt = 300 \; rac{-\cos(5x+\omega t)}{\omega} + C(x) = -rac{300}{1.5 imes 10^9} \cos(5x+\omega t) \, dt$$

Since the wave has no static (*t*–independent) magnetic part we take C(x) = 0,

giving

$$B_y(x,t) = rac{300}{1.5 imes 10^9} {
m cos}(5x+1.5 imes 10^9 t) = 2 imes 10^{-7} {
m cos}(5x+1.5 imes 10^9 t) ~{
m T} \,.$$

6. Check amplitude by $E_0/B_0=c.$ $E_0=60$ V/m and $cpprox 3 imes 10^8$ m/s give

$$B_0 = rac{E_0}{c} = rac{60}{3 imes 10^8} = 2 imes 10^{-7}\,{
m T},$$

in agreement with the above.

Therefore the magnetic field is

$$B_y = 2 \times 10^{-7} \cos(5x + 1.5 \times 10^9 t) \text{ T},$$

 \wedge

which corresponds to choice (1).

10 Problem 10

A ball of mass 0.5 kg is dropped from a height of 40 m. The ball hits the ground and rises to a height of 10 m. The impulse imparted to the ball during its collision with the ground is (Take $g = 9.8 \text{ m/s}^2$)



Hide Solution

Solution:

Here is a detailed step-by-step solution, with inline math in `...` and display math in `

. . .

1. Determine the speed just before impact.

The ball is dropped from rest from height $h_1 = 40$ m, so by energy conservation (or $v^2 = u^2 + 2gh$ with u = 0):

$$v_{
m before} = \sqrt{2gh_1} = \sqrt{2\cdot 9.8\cdot 40} = \sqrt{784} = 28 \ {
m m/s}$$

(directed downward).

2. Determine the speed just after rebounding. The ball rises to height $h_2 = 10$ m, so its rebound speed v_{after} satisfies

$$v_{
m after} = \sqrt{2gh_2} = \sqrt{2\cdot 9.8\cdot 10} = \sqrt{196} = 14 \ {
m m/s}$$

(directed upward).

3. Compute the change in momentum (impulse).

Take upward as the positive direction. Just before impact the momentum is

$$p_{ ext{before}} = m \, v_{ ext{before}} = 0.5 \, (-28) = -14 ext{ kg} \cdot ext{m/s},$$

and just after,

$$p_{
m after} = m \, v_{
m after} = 0.5 \, (+14) = +7 \; {
m kg} \cdot {
m m/s}.$$



Solution:

Here is a step-by-step solution. We take the direction of the current from A to B as shown, and we will write down the drops across the inductor, the 5 V source, and the 2 Ω resistor, and then use KVL to find

$$V_A - V_B$$
.

1. Label the instantaneous current and its rate of change

$$i=2~\mathrm{A}, \qquad rac{di}{dt}=1~\mathrm{A/s}.$$

2. Voltage drop across the inductor (using passive-sign convention, i.e.\ current enters the "+" terminal):

$$v_L = L \, rac{di}{dt} = 1 \, \mathrm{H} imes 1 \, rac{\mathrm{A}}{\mathrm{s}} = 1 \, \mathrm{V}.$$

By convention this is a drop of 1 V in the direction of the current.

3. Voltage drop across the 5 V source. From the diagram the current enters the positive plate of the 5 V battery and leaves its negative plate, so in the direction of the current there is a 5 V drop:

$$v_{\mathrm{batt}} = 5 \mathrm{V}.$$

4. Voltage drop across the resistor (Ohm's law):

$$v_R = i\,R = 2~\mathrm{A} imes 2~\Omega = 4~\mathrm{V}.$$

5. Now apply KVL from A to B (sum of drops = total $V_A - V_B$):

$$V_A - V_B \;=\; v_L + v_{
m batt} + v_R \;=\; 1 \;+\; 5 \;+\; 4 \;=\; 10 \; {
m V} \,.$$

~

Answer: $V_A - V_B = 10$ V.

12 Problem 12



Solution:

Here is the step-by-step solution.

1. The magnetic moment μ of a single circular current loop of radius r carrying current I is

$$\mu ~=~ I \cdot A ~=~ I \left(\pi r^2
ight).$$

2. For two coils with the same current I but radii r_1 and r_2 , the ratio of their magnetic moments is

$$rac{\mu_1}{\mu_2} = rac{I\,\pi\,r_1^2}{I\,\pi\,r_2^2} = rac{r_1^2}{r_2^2}\,.$$

3. Since the radii are in the ratio $r_1 : r_2 = 1 : 2$, we have

$$rac{\mu_1}{\mu_2} = rac{1^2}{2^2} = rac{1}{4} \, ,$$

i.e.\ $\mu_1: \mu_2 = 1:4.$

Answer: 1:4.

13 Problem 13

In a certain camera, a combination of four similar thin convex lenses are arranged axially in contact. Then the power of the combination and the total magnification in comparison to the power (p) and magnification (m) for each lens will be, respectively -

(1) 4p and 4m

(2) p⁴ and 4m

(3) 4p and m⁴

Hide Solution

Solution:

Here is a detailed solution.

(1) Power of four thin lenses in contact

If you have thin lenses of powers p_1,\ldots,p_n in contact, the resulting power is the algebraic sum

$$P_{\mathrm{eff}} = p_1 + \cdots + p_n.$$

Since all four lenses are identical of power p, the net power is

$$P_{\mathrm{eff}} = p + p + p + p = 4p$$
.

Hence the equivalent focal length is

$$f_{
m eff} = rac{1}{P_{
m eff}} = rac{1}{4p}$$

(2) Magnification of a camera lens under extension

In a simple camera (or a macro-tube), when you focus on a close object you move the lens away from the film by some extension Δ . For a single thin lens of focal length f, one finds (for small magnification)

$$m ~pprox ~rac{\Delta}{f}$$
 .

That is, the linear magnification is proportional to the extension Δ and inversely proportional to the focal length.

Now if you replace that single lens by the four-lens combination, its focal length becomes

^

$$f_{
m eff} = rac{1}{4p} = rac{f}{4} \quad ig(f=1/pig).$$

But the camera extension Δ is unchanged (you still move the lens group the same distance Δ from the film). Hence the new magnification is

$$m_{
m eff}~pprox~rac{\Delta}{f_{
m eff}}~=~rac{\Delta}{f/4}~=~4~rac{\Delta}{f}~=~4\,m.$$

(3) Final answer

Thus the combination of four identical thin convex lenses in contact has power 4p and gives, for the same camera extension, a magnification 4m. In other words, choice (1):

and

4p

4m

 $\overline{}$

14 Problem 14

An oxygen cylinder of volume 30 litre has 18.20 moles of oxygen. After some oxygen is withdrawn from the cylinder, its gauge pressure drops to 11 atmospheric pressure at temperature 27°C. The mass of the oxygen withdrawn from the cylinder is nearly equal to: (Given, R = 100/12 J mol⁻¹ K⁻¹, molecular mass of $O_2 = 32$, 1 atm pressure = 1.01 × 10⁵ N/m²)

(1) 0.125 kg

(2) 0.144 kg

(3) 0.116 kg

Hide Solution

Solution:

Here is a detailed step-by-step solution. We shall denote gauge pressures by $P_{
m gauge}$ and absolute pressures by P.

Given data

- Volume of cylinder:

 $V = 30 \text{ L} = 30 \times 10^{-3} \text{ m}^3 = 0.03 \text{ m}^3.$

- Initial number of moles of O₂:

- Final gauge pr

- Temperature (

 $T = 27^{\circ}C = 27 + 273 = 300$ K.

- Gas constant (given):

$$R = rac{100}{12}\,\mathrm{J\,mol^{-1}K^{-1}} pprox 8.333\,\mathrm{J\,mol^{-1}K^{-1}}.$$

- Molecular mass of O₂:

$$n_1 = 18.20 ext{ mol.}$$

ressure (as read on the cylinder):
 $P_{ ext{gauge},2} = 11 ext{ atm.}$
constant):

 $M = 32 \,\mathrm{g/mol.}$ - 1 atm in SI units: $1 \operatorname{atm} = 1.01 \times 10^5 \operatorname{Pa.}$ **Step 1: Find the initial absolute pressure P_1 .** By the ideal-gas law, $P_1V = n_1RT \implies P_1 = rac{n_1RT}{V}$ Substitute: $P_1 = \frac{(18.20)\left(\frac{100}{12}\right)(300)}{0.03}$ 18.20 imes 8.333 imes 300 $r=1.517 imes 10^6\,{
m Pa}pprox 15.0~{
m atm.}$ 0.03 **Step 2: Convert the final gauge pressure to absolute pressure.** The gauge reads pressure above atmospheric, so $P_2 = P_{
m atm} + P_{
m gauge,2} = 1 \; {
m atm} + 11 \; {
m atm} = 12 \; {
m atm} = 12 imes 1.01 imes 10^5 \, {
m Pa} = 1.212 imes$

Step 3: Find the final number of moles n_2 . Again by PV = nRT,

$$n_2 = rac{P_2 V}{RT} = rac{\left(1.212 imes 10^6
ight) \left(0.03
ight)}{\left(100/12
ight) \left(300
ight)} = 14.56 ext{ mol.}$$

Step 4: Compute the moles withdrawn and then the mass. Moles withdrawn:

 $\Delta n = n_1 - n_2 = 18.20 - 14.56 = 3.64 \text{ mol.}$

Mass withdrawn:

$$m = \Delta n imes rac{M}{1000} = 3.64 imes rac{32}{1000} = 0.1165 ext{ kg} pprox 0.116 ext{ kg}.$$

Answer.

The mass of oxygen withdrawn is nearly 0.116\,kg, which corresponds to choice (3).

15 Problem 15

In some appropriate units, time (t) and position (x) relation of a moving particle is given by $t = x^2 + x$. The acceleration of the particle is

(1) $-2/(x+2)^3$

(2) $-2/(2x+1)^3$

(3) +2 /
$$(x + 1)^3$$

(4)
$$+2/(2x+1)$$

Hide Solution

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Solution:

Here is a detailed solution. We are given the "time-position" relation

$$t = x^2 + x$$

and asked to find the acceleration

$$a=rac{d^2x}{dt^2}.$$



Step 3. Differentiate the velocity with respect to time to get the acceleration:

$$a = rac{d^2 x}{dt^2} = rac{d}{dt} \Big(rac{dx}{dt}\Big) = rac{d}{dx} \Big(rac{dx}{dt}\Big) \; rac{dx}{dt}$$

(by the chain rule).

We have

$$\frac{dx}{dt} = \frac{1}{2x+1} \quad \Longrightarrow \quad \frac{d}{dx} \left(\frac{dx}{dt}\right) = \frac{d}{dx} \left((2x+1)^{-1}\right) = -1\left(2x+1\right)^{-2} \cdot 2 = -\frac{1}{(2x+1)^{-2}} \cdot 2 = -\frac{1}{(2x+1)^{-2$$

Hence

$$a=-rac{2}{(2x+1)^2}\,rac{1}{2x+1}=-rac{2}{(2x+1)^3}.$$

Therefore the acceleration is

$$\boxed{a=-\frac{2}{(2x+1)^3}},$$

which corresponds to choice (2).

Problem 16 16 To an ac power supply of 220 V at 50 Hz, a resistor of 20 Ω , a capacitor of reactance 25 Ω and an inductor of reactance 45 Ω are connected in series. The corresponding current in the circuit and the phase angle between the current and the voltage is, respectively -7.8 A and 30° (1) 7.8 A and 45° (2) 15.6 A and 30° (3) 15.6 A and 45° (4) Hide Solution

Solution:

Here is a detailed step-by-step solution using TeX delimiters.

Given:

An AC source of rms voltage $V=220\,\mathrm{V}$ at $50\,\mathrm{Hz}$ drives a series circuit of

– a resistor $R=20~\Omega$,

– a capacitive reactance $X_C=25~\Omega$,

– an inductive reactance $X_L=45~\Omega.$

We wish to find

1. The rms current I in the circuit.

2. The phase angle ϕ between the applied voltage and the current.

Step 1: Write the total impedance. For a series RLC circuit the impedance is

$$Z \;=\; R \;+\; j \left(X_L - X_C
ight).$$

Substitute the given values:

$$Z = 20 + j (45 - 25) = 20 + j 20 \quad (\Omega).$$

Step 2: Compute the magnitude of the impedance.

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{20^2 + 20^2} = \sqrt{400 + 400} = \sqrt{800} = 20\sqrt{2}$$

Step 3: Find the current. Ohm's law for rms quantities gives

$$I = rac{V}{|Z|} = rac{220}{20\sqrt{2}} = rac{11}{\sqrt{2}} pprox 7.78 \, \mathrm{A} \ pprox 7.8 \, \mathrm{A}.$$

Step 4: Determine the phase angle. The phase angle ϕ (voltage minus current) is

$$\phi ~=~ rctanigg(rac{X_L-X_C}{R}igg) ~=~ rctanigg(rac{20}{20}igg) ~=~ rctan(1) ~=~ 45^\circ.$$

Since $X_L > X_C$, the circuit is net inductive and the current **lags** the voltage by 45° .

Answer:

The rms current is Ipprox 7.8 A, and the current lags the voltage by $\phi=45^\circ.$

This corresponds to choice **(2)**.



The Sun rotates around its centre once in 27 days. What will be the period of revolution if the Sun were to expand to twice its present radius without any external influence? Assume the Sun to be a sphere of uniform density.

 $\overline{}$



Solution:

Here is a step-by-step solution using conservation of angular momentum. We treat the Sun as a uniform solid sphere of mass M.
Step 1. Write down the moment of inertia of a uniform sphere of radius R

$$I=rac{2}{5}\,M\,R^2$$

Step 2. Initial quantities.

Let the initial radius be R_1 , the initial moment of inertia

$$I_1 = rac{2}{5} M \, R_1^2,$$

and the initial rotation period be $T_1=27\,\mathrm{days}.\,$ The corresponding angular speed is

 $\omega_1 = rac{2\pi}{T_1}$

Step 3. After expansion to twice the radius,

so the new moment of inertia is

$$I_2 = rac{2}{5}MR_2^2 = rac{2}{5}M(2R_1)^2 = 4\;rac{2}{5}MR_1^2 = 4\,I_1.$$

 $R_2 = 2 \, R_1$

Step 4. Conservation of angular momentum (no external torque)

$$L_{ ext{initial}} = L_{ ext{final}} \quad \Longrightarrow \quad I_1 \, \omega_1 = I_2 \, \omega_2.$$

Hence

$$\omega_2=rac{I_1}{I_2}\,\omega_1=rac{1}{4}\,\omega_1.$$

Step 5. Compute the new rotation period T_2 . Since

$$T = \frac{2\pi}{\omega},$$

we have

$$T_2 = rac{2\pi}{\omega_2} = rac{2\pi}{\omega_1/4} = 4 \ rac{2\pi}{\omega_1} = 4 \ T_1 = 4 imes 27 \ {
m days} = 108 \ {
m days}$$

Answer: 108 days. (Option 4)

18 Problem 18

A model for quantized motion of an electron in a uniform magnetic field B states that the flux passing through the orbit of the electron is n(h/e) where n is an integer, h is Planck's constant and e is the magnitude of electron's charge. According to the model, the magnetic moment of an electron in its lowest energy state will be (m is the mass of the electron)

(1) (he) / (π m)

(2) (he) / (2 π m)

(3) (heB) / (π m)

(4) (heB) / (2 π m)

Hide Solution

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Solution:

Here is a detailed step-by-step solution using the given flux-quantization model.

1. Flux quantization

The model states that the magnetic flux through the electron's circular orbit of area ${\boldsymbol A}$ is

$$\Phi \;=\; B \,A \;=\; n \, {h \over e} \,, \qquad n = 1, 2, 3, \ldots$$

Hence

$$A = \frac{n h/e}{B}.$$

2. Relate the orbital frequency to B

For an electron of mass m moving in a circle of radius r in a field B, the centripetal force is provided by the magnetic Lorentz force:

$$\frac{mv^2}{r} = evB \implies v = \frac{eBr}{m}$$

The angular frequency is $\omega=v/r$, so

$$\omega = \frac{eB}{m}$$

3. Compute the orbital current IThe electron makes one revolution in time $T=2\pi/\omega$, so the current is

$$I = \frac{e}{T} = e \frac{\omega}{2\pi} = \frac{e}{2\pi} \frac{eB}{m} = \frac{e^2B}{2\pi m}$$

4. Magnetic moment μ By definition,

$$\mu \;=\; I\,A \;=\; rac{e^2B}{2\pi m} \; imes\; rac{n\,h/(e)}{B} \;=\; rac{n\,e\,h}{2\pi m}\,.$$

In the lowest energy state n = 1, this becomes

$$\mu = rac{e \, h}{2 \pi m}$$

hе

Match to the given choices
 The above result corresponds to choice (2)

19 Problem 19

Three identical heat conducting rods are connected in series as shown in the figure. The rods on the sides have thermal conductivity 2K while that in the middle has thermal conductivity K. The left end of the combination is maintained at temperature 3T and the right end at T. The rods are thermally insulated from outside. In steady state, temperature at the left junction is T_1 and that at the right junction is T_2 . The ratio T_1 / T_2 is



(1) 3/2
(2) 4/3
(3) 5/3
(4) 5/4
THide Solution
Solution: Here is a complete step-by-step solution. We assume each rod has length L and cross-sectional area A . 1. Thermal resistance of a rod of conductivity k is $R = \frac{L}{kA}.$
2. Thus the three rods have resistances $R_1=rac{L}{2KA}, \qquad R_2=rac{L}{KA}, \qquad R_3=rac{L}{2KA}.$
3. Since they are in series, the total resistance is $R_{\rm tot}=R_1+R_2+R_3=\frac{L}{2KA}+\frac{L}{KA}+\frac{L}{2KA}=\frac{2L}{KA}.$

4. The end-temperatures are $T_{
m left}=3T$ and $T_{
m right}=T$, so the total heat current in steady state is

$$Q = rac{T_{
m left} - T_{
m right}}{R_{
m tot}} = rac{3T - T}{2L/(KA)} = rac{2T}{2L/(KA)} = rac{KA}{L} \, T \, .$$

5. The temperature drop across the first rod is

$$\Delta T_1 = Q R_1 = \left(\frac{KA}{L}T\right) \left(\frac{L}{2KA}\right) = \frac{T}{2}$$

Hence the junction temperature T_1 (measured from the left end at 3T) is

$$T_1 = 3T - \Delta T_1 = 3T - \frac{1}{2}T = \frac{5T}{2}$$

6. The drop across the second (middle) rod is

$$\Delta T_2 = Q R_2 = \left(\frac{KA}{L}T\right)\left(\frac{L}{KA}\right) = T,$$

so the right-junction temperature is

$$T_2 = T_1 - \Delta T_2 = rac{5T}{2} - T = rac{3T}{2} \,.$$

7. Therefore the required ratio is

$$rac{T_1}{T_2} = rac{rac{5T}{2}}{rac{3T}{2}} = rac{5}{3}$$

Answer:
$$rac{T_1}{T_2}=5/3$$
 .

20 Problem 20

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The plates of a parallel plate capacitor are separated by d. Two slabs of different dielectric constant K_1 and K_2 with thickness $\frac{3}{8}d$ and $\frac{d}{2}$, respectively are inserted in the capacitor. Due to this, the capacitance becomes two times larger than when there is nothing between the plates. If $K_1 = 1.25K_2$, the value of K_1 is:



Hide Solution

Solution:

Here is a detailed solution. We shall treat the region between the plates as three dielectrics in series:

– a slab of thickness $t_1=rac{3}{8}d$ and dielectric constant K_1 ,

- a slab of thickness $t_2 = rac{1}{2}d$ and dielectric constant K_2 ,
- the remaining gap of thickness

$$t_3 = d - \left(\frac{3}{8}d + \frac{1}{2}d\right) = d - \frac{7}{8}d = \frac{1}{8}d$$

which is vacuum ($K_3 = 1$).

1. Capacitance with nothing between the plates (empty capacitor) is

$$C_0 = rac{arepsilon_0 A}{d}$$
 .

2. When the three layers are inserted in series, the total capacitance C satisfies

$$\frac{1}{C} = \frac{t_1}{\varepsilon_0 K_1 A} + \frac{t_2}{\varepsilon_0 K_2 A} + \frac{t_3}{\varepsilon_0 \cdot 1 \cdot A} = \frac{1}{\varepsilon_0 A} \Big(\frac{t_1}{K_1} + \frac{t_2}{K_2} + t_3 \Big).$$

3. We are told that the new capacitance is twice the empty-capacitance,

$$C = 2 C_0 \implies \frac{1}{O} = \frac{1}{2C_0} = \frac{1}{2} \frac{d}{\varepsilon_0 A}$$

Hence

$$rac{1}{arepsilon_0 A} \Big(rac{t_1}{K_1} + rac{t_2}{K_2} + t_3 \Big) \ = \ rac{d}{2 \, arepsilon_0 A} \quad \Longrightarrow \quad rac{t_1}{K_1} + rac{t_2}{K_2} + t_3 \ = \ rac{d}{2 \, arepsilon_0 A}$$

4. Substitute $t_1=rac{3}{8}d$, $t_2=rac{1}{2}d$, $t_3=rac{1}{8}d$ and divide through by d:

$$rac{3/8}{K_1} \ + \ rac{1/2}{K_2} \ + \ rac{1}{8} \ = \ rac{1}{2}$$

5. Use the given relation $K_1=1.25\,K_2=rac{5}{4}\,K_2$, so $K_2=rac{4}{5}\,K_1$. Then

$$\frac{3}{8\,K_1} \ + \ \frac{1}{2\,K_2} \ + \ \frac{1}{8} \ = \ \frac{3}{8K_1} \ + \ \frac{1}{2\,(4/5\,K_1)} \ + \ \frac{1}{8} \ = \ \frac{3}{8K_1} \ + \ \frac{5}{8K_1} \ + \ \frac{1}{8} \ = \ \frac{8}{8K_1}$$

Hence

$$rac{1}{K_1} = rac{1}{2} - rac{1}{8} = rac{4-1}{8} = rac{3}{8} \implies K_1 = rac{8}{3} pprox 2.66.$$

Answer:
$$K_1=rac{8}{3}pprox 2.66.$$

21 Problem 21

Two cities X and Y are connected by a regular bus service with a bus leaving in either direction every T min. A girl is driving scooty with a speed of 60 km/h in the direction X to Y notices that a bus goes past her every 30 minutes in the direction of her motion, and every 10 minutes in the opposite direction. Choose the correct option for the period T of the bus service and the speed (assumed constant) of the buses.

(1) 9 min, 40 km/h

(2) 25 min, 100 km/h

(3) 10 min, 90 km/h

(4) 15 min, 120 km/h

Hide Solution

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Solution:

Here is a step-by-step solution.

1. Notation

Let

u=60 km/h be the scooty's speed,

v km/h the (unknown) bus speed,

T minutes the bus-departure interval in each direction.

It is convenient to set

$$D = rac{T}{60} \quad ext{(hours between successive departures)}.$$

2. Distance between successive buses

Since each bus travels at speed v, the distance (along the road) between two consecutive buses in one direction is

 $\Delta = v D$ (km).

3. Overtakes (same-direction encounters)

The relative speed of a bus overtaking the scooty is v - u. We are told an overtake occurs every 30 min = 0.5 h, so

$$\frac{\Delta}{v-u} = 0.5 \implies \frac{vD}{v-60} = 0.5.$$
 (1)

4. Meetings (opposite-direction encounters)

The relative speed of an oncoming bus and the scooty is v + u. We are told a meeting occurs every 10 min = 1/6 h, so

$$\frac{\Delta}{v+u} = \frac{vD}{v+60} = \frac{1}{6}.$$
(2)

5. Solve the two equations From (1):

$$v\,D=0.5\,(v-60)\implies vD-0.5v=-30\implies v\,(D-0.5)=-30\implies v=rac{1}{0.5}$$

From (2):

$$v D = rac{1}{6} \left(v + 60
ight) \implies v D - rac{1}{6} v = 10 \implies v \left(D - rac{1}{6}
ight) = 10 \implies v = rac{10}{D - rac{1}{6}}$$

Equate the two expressions for v:

$$\frac{30}{0.5 - D} = \frac{10}{D - \frac{1}{6}} \implies 30(D - \frac{1}{6}) = 10(0.5 - D) \implies 40D = 10 \implies D = -10$$

Hence

$$T = 60 D = 60 \cdot \frac{1}{4} = 15$$
 minutes,

and then

$$v=rac{30}{0.5-0.25}=rac{30}{0.25}=120~{
m km/h}$$

6. Conclusion

The bus-departure interval is $T=15~{\rm min},$ and the bus speed is $120~{\rm km/h}.~{\rm This}$ is choice (4).

A uniform rod of mass 20 kg and length 5 m leans against a smooth vertical wall making an angle of 60° with it. The other end rests on a rough horizontal floor. The friction force that the floor exerts on the rod is (take $g = 10 \text{ m/s}^2$)

(1) 100 N
(2) $100\sqrt{3}$ N
(3) 200 N
(4) $200\sqrt{3}$ N
Hide Solution
Solution:
Here is a complete solution:
1. Geometry The rod makes an angle of 60° with the vertical wall, so its angle with the horizontal floor is
$lpha \;=\; 90^\circ - 60^\circ \;=\; 30^\circ .$

2. Free-body diagram

At the lower end (point A) there are

- a vertical normal reaction \boldsymbol{N} from the floor,
- a horizontal frictional force ${\cal F}$ from the floor (towards the wall).

At the upper end (point B) the smooth wall exerts only a horizontal reaction ${\boldsymbol R}$

(towards the rod).

The weight W = mg acts downward at the midpoint of the rod.

- 3. Equations of equilibrium
 - (a) \sum vertical forces = 0:

$$N - mg = 0 \implies N = mg = 20 \cdot 10 = 200 \text{ N}.$$

(b) \sum horizontal forces = 0:

$$R-F=0 \implies F=R.$$

(c) \sum moments about the lower end A = 0. Taking counter-clockwise as positive, the moment of R (at height $\ell \sin \alpha$) balances the moment of mg (at horizontal lever arm $\frac{\ell}{2} \cos \alpha$):

$$R\left(\ell\sin\alpha\right) \ - \ mg\left(\frac{\ell}{2}\cos\alpha\right) \ = \ \mathbf{0} \ \implies \ R = \frac{mg\left(\frac{1}{2}\cos\alpha\right)}{\sin\alpha} = \frac{mg\cos\alpha}{2\sin\alpha} = \frac{mg\cos\alpha}{2\tan\alpha}$$

4. Compute FSince F = R, and mg = 200 N, $\alpha = 30^{\circ}$,

$$F = rac{200}{2 an 30^\circ} = rac{200}{2 \left(1/\sqrt{3}
ight)} = 100 \sqrt{3} \ {
m N} \, .$$

Answer: the frictional force at the floor is $100\sqrt{3}$ N (option 2).

23 Problem 23

In an oscillating spring mass system, a spring is connected to a box filled with sand. As the box oscillates, sand leaks slowly out of the box vertically so that the average frequency $\omega(t)$ and average amplitude A(t) of the system change with time t. Which one of the following options schematically depicts these changes correctly?







Hide Solution

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Solution:

Here is a convenient way to see what happens. Take the box + sand as a onedimensional spring–mass system oscillating horizontally on a frictionless support, with the sand leaking out vertically (so there is no horizontal impulse when a bit of sand leaves). Then the instantaneous equation of motion of the remaining mass m(t) is

$$m(t)\,\ddot{x}+k\,x=0.$$

1. Instantaneous angular frequency By inspection one has

$$\omega(t) \;=\; \sqrt{rac{k}{m(t)}}\,.$$

As sand leaks out m(t) decreases, so $\omega(t)$ increases with time.

2. Energy balance and amplitude The mechanical energy of the remaining mass + spring is

$$E(t) \;=\; rac{1}{2} \, m(t) \, \dot{x}^2 \;+\; rac{1}{2} \, k \, x^2 \,.$$

Differentiate:

$$\frac{dE}{dt} = \frac{1}{2}\dot{m}(t)\dot{x}^2 + m\dot{x}\ddot{x} + kx\dot{x} = \frac{1}{2}m'(t)\dot{x}^2$$
 (since $m\ddot{x} + kx = 0$).

Because m'(t) < 0 and $\dot{x}^2 > 0$, dE/dt < 0: the oscillator loses energy as sand carrying horizontal kinetic energy falls away.

Now over one cycle of slow leakage we may replace \dot{x}^2 by its average $\frac{1}{2}A^2\omega^2.$ Thus

$$ig\langle dE/dtig
angle = rac{1}{2}\,m'\,rac{1}{2}\,A^2\,\omega^2 = rac{1}{4}\,m'\,A^2rac{k}{m} = rac{1}{2}\,rac{m'}{m}\,E$$

Hence

$$rac{dE}{E} = rac{1}{2} \, rac{dm}{m} \implies E(t) \propto \sqrt{m(t)} \implies A^2 \propto E \propto \sqrt{m(t)} \implies A(t) \propto m(t)$$

Since m(t) is decreasing, A(t) also decreases (though more slowly than m).

- 3. Conclusion
 - + $\omega(t)=\sqrt{k/m(t)}
 ightarrow \omega$ rises as m falls.
 - $A(t) \propto m^{1/4} o A$ falls as m falls.

Graphically this is exactly choice (2): an increasing (concave-down) $\omega(t)$ and a decreasing (concave-down) A(t).

24 Problem 24

A balloon is made of a material of surface tension S and its inflation outlet (from where gas is filled in it) has small area A. It is filled with a gas of density ρ and takes a spherical shape of radius R. When the gas is allowed to flow freely out of it, its radius r changes from R to 0 (zero) in time T. If the speed v(r) of gas coming out of the balloon depends on r as r^a and $T \propto S^{\alpha} A^{\beta} \rho^{\gamma} R^{\delta}$ then

(1)
$$a = \frac{1}{2}, \alpha = \frac{1}{2}, \beta = -1, \gamma = +\frac{1}{2}, \delta = \frac{3}{2}$$

(2)
$$a = -rac{1}{2}, lpha = -rac{1}{2}, eta = -1, \gamma = -rac{1}{2}, \delta = rac{5}{2}$$

(3)
$$a = \frac{1}{2}, \alpha = -\frac{1}{2}, \beta = -1, \gamma = \frac{1}{2}, \delta = \frac{7}{2}$$

(4)
$$a = \frac{1}{2}, \alpha = \frac{1}{2}, \beta = -\frac{1}{2}, \gamma = \frac{1}{2}, \delta = \frac{7}{2}$$

Hide Solution

Solution:

Here is a complete step-by-step derivation. In what follows we denote by r the instantaneous radius of the balloon, and by v(r) the speed at which the gas issues from the small outlet of area A.

1) Laplace's law for a single soap-film (or thin-membrane) sphere gives a pressure jump

$$\Delta P = P_{
m in} - P_{
m out} = rac{2\,S}{r}$$

2) By Bernoulli's theorem (work done by the pressure drop goes into kinetic energy of the jet)

$$rac{1}{2}
ho v^2 = \Delta P \implies v(r) = \sqrt{rac{2 \,\Delta P}{
ho}} = \sqrt{rac{4 \,S}{
ho \,r}} \propto r^{-rac{1}{2}}$$

Hence

$$a = -\frac{1}{2}.$$

3) Conservation of volume (or mass, since the gas density in the jet is the same ρ) gives

$$rac{dV}{dt}=-A\,v(r),\qquad V(r)=rac{4\pi}{3}\,r^3.$$

Hence

$$rac{d}{dt} \Big(rac{4\pi}{3} \, r^3 \Big) = 4\pi \, r^2 \, \dot{r} \; = \; -A \, C \, r^a,$$

where $v(r)=C\,r^a.$ Thus

$$\dot{r} = -rac{A\,C}{4\pi} \; r^{\,a-2},$$

or

$$\int_{R}^{0} r^{2-a}\,dr \;=\; -rac{A\,C}{4\pi}\,\int_{0}^{T}dt \implies T = rac{4\pi}{A\,C\,(3-a)}\;R^{\,3-a}.$$

 $\delta = 3 - a.$

Therefore

4) Finally, from step 2 we had

$$C = \sqrt{rac{4\,S}{
ho\,r}} \ / \ r^a \ \propto \ S^{1/2} \,
ho^{-1/2} \, ,$$

SO

$$T \,\propto\, rac{1}{C} \,R^{3-a} \,\propto\, S^{-1/2} \,A^{-1} \,
ho^{+1/2} \,R^{\,3-a}.$$

Putting in $a=-rac{1}{2}$ gives

$$\delta = 3 - (-rac{1}{2}) = rac{7}{2}, \quad lpha = -rac{1}{2}, \quad eta = -1, \quad \gamma = +rac{1}{2}.$$

In conclusion one finds

$$v(r) \propto r^{-1/2}, ~~T~\propto~S^{-1/2}~A^{-1}~
ho^{+1/2}~R^{7/2},$$

so that

25 Problem 25

Consider the diameter of a spherical object being measured with the help of a Vernier callipers. Suppose its 10 Vernier Scale Divisions (V.S.D.) are equal to its 9 Main Scale Divisions (M.S.D.). The least division in the M.S. is 0.1 cm and the zero of V.S. is at x = 0.1 cm when the jaws of Vernier callipers are closed. If the main scale reading for the diameter is M = 5 cm and the number of coinciding vernier division is 8, the measured diameter after zero error correction, is

(1) 5.18 cm

(2) 5.08 cm

(3) 4.98 cm

(4) 5.00 cm

Hide Solution

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Solution:

Here is a step-by-step solution in LaTeX form:

- 1. Determine the value of one Vernier-scale division (VSD) and the least count (LC).
 - One main-scale division (MSD) = 0.1 cm.
 - Given $10\,\text{VSD}=9\,\text{MSD}=9\times0.1\,\text{cm}=0.9\,\text{cm},$ so

$$1 \text{ VSD} = \frac{0.9 \text{ cm}}{10} = 0.09 \text{ cm}.$$

- The least count is

LC = 1 MSD - 1 VSD = 0.1 cm - 0.09 cm = 0.01 cm.

2. Find the zero-error correction.

– When the jaws are closed, the zero of the vernier lies at x = 0.1 cm on the main scale.

– Thus the instrument reads +0.1 cm when it should read 0.

- Zero error = +0.1 cm (to be subtracted from any reading).

3. Compute the raw reading for the diameter.

- Main-scale reading: $M=5.00\,{
 m cm}.$
- Number of coinciding vernier divisions: N = 8.
- Vernier contribution $= N imes \mathrm{LC} = 8 imes 0.01 = 0.08$ cm.
- Raw reading $= M + N \cdot \mathrm{LC} = 5.00 + 0.08 = 5.08\,\mathrm{cm}.$
- 4. Apply the zero-error correction.

True diameter = $(M + N \cdot \text{LC}) - (\text{zero error}) = 5.08 \text{ cm} - 0.10 \text{ cm} = 4.98 \text{ cm}.$

Answer: 4.98 cm (option 3).

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Problem Solutions

View detailed solutions to each problem. Click on "Show Solution" to see the full explanation.



Solution:

We apply the Ampère–Maxwell law in integral form to a circular Amperian loop of radius r, coaxial with the plates. Since there is no conduction current between the plates, only the displacement-current term contributes:

$$\oint_C {f B} \cdot d{m \ell} \;=\; \mu_0 ackslash {f eps}_0 \, {d\over dt} \int_S {f E} \cdot d{f A} \;.$$

Here S is the flat disk of radius r bounded by the loop C.

1. Electric field between the plates. If the surface charge density on the plates is $\sigma(t)$, then the field in the gap is

$$E(t) = rac{\sigma(t)}{\langle ext{eps}_0},$$

assumed uniform over the plate area.

2. Case 1: $r \leq a$, where a is the plate radius. The electric flux through S is

$$\Phi_E(r) = \int_S E \, dA = E(t) \, \pi r^2 = rac{\sigma(t)}{igvee ext{eps}_0} \, \pi r^2 \, .$$

Hence

$$rac{d\Phi_E}{dt} = rac{\pi r^2}{igvee \mathbf{ps}_0} \, rac{d\sigma}{dt}$$

and the Ampère-Maxwell law gives

$$\oint_C B\,dl = \mu_0ackslash {f eps}_0 \; {d\Phi_E\over dt} = \mu_0\,\pi r^2\,{d\sigma\over dt}.$$

By symmetry B is tangential and constant on C, so

$$B(r)\left(2\pi r
ight)=\mu_{0}\,\pi r^{2}\,rac{d\sigma}{dt}\quad\Longrightarrow\quad B(r)=rac{\mu_{0}\,r}{2}\,rac{d\sigma}{dt}\qquad(r\leq a).$$

3. Case 2: $r \ge a$.

Now the field exists only over the disk of radius a, so

$$\Phi_E = \pi a^2 \, rac{\sigma(t)}{\langle \mathbf{eps}_0}, \qquad rac{d\Phi_E}{dt} = rac{\pi a^2}{\langle \mathbf{eps}_0} \, rac{d\sigma}{dt},$$

 $\oint_C B\, dl = \mu_0 \,\pi a^2 \, {d\sigma \over dt},$

giving

and

$$B(r)\left(2\pi r
ight)=\mu_{0}\pi a^{2}rac{d\sigma}{dt} \implies B(r)=rac{\mu_{0}a^{2}}{2r}rac{d\sigma}{dt} \qquad (r\geq a).$$

4. Summary of B(r):

$$B(r)=rac{\mu_0}{2} \; rac{d\sigma}{dt} iggl\{ egin{array}{cc} r, & r\leq a, \ a^2/r, & r\geq a. \end{array} iggr]$$

This function grows linearly from r = 0 to r = a, reaches its maximum at r = a, and then falls off as 1/r for r > a. Hence the magnetic field due solely to the displacement current is nonzero both inside and outside the plates, with its maximum on the "imaginary" cylindrical surface of radius *a* joining the plate edges.

Answer: choice (3).

An unpolarized light beam travelling in air is incident on a medium of refractive index 1.73 at Brewster's angle. Then-

(1) Reflected light is completely polarized and the angle of reflection is close to 60°

- (2) Reflected light is partially polarized and the angle of reflection is close to 30°
- (3) Both reflected and transmitted light are perfectly polarized with angles of reflection and refraction close to 60° and 30° , respectively.
- (4) Transmitted light is completely polarized with angle of refraction close to 30°

Hide Solution

Solution:

Here is a step-by-step solution:

1. Identify the refractive indices and Brewster's angle condition.

We have light going from air ($n_1 = 1.00$) into a medium of index $n_2 = 1.73$. Brewster's angle θ_B satisfies

$$an heta_B = rac{n_2}{n_1}\,.$$

2. Compute θ_B .

$$heta_B = rctan(1.73) pprox 60^\circ$$

3. Angle of reflection.

By the law of reflection,

$$heta_{
m r}= heta_{
m i}= heta_Bpprox 60^\circ$$

4. Polarization of the reflected beam at Brewster's angle.

- The Fresnel reflection coefficient for light polarized **parallel** to the plane of incidence goes to zero at θ_B :

$$r_p=0 \quad ({
m at} \ heta_i= heta_B) \,.$$

- Only the **perpendicular**-polarized (s-polarized) component is reflected.
- ⇒ The **reflected light** is **completely polarized**.
- 5. Angle of refraction and polarization of the transmitted beam. Snell's law gives

$$n_1 \sin heta_i = n_2 \sin heta_t \implies \sin heta_t = rac{1}{1.73} \sin 60^\circ = 0.5 \implies heta_t = 30^\circ \,.$$

Both s– and p–components are transmitted (with different amplitudes), so the **transmitted light** is only **partially polarized**.

- 6. Conclusion.
 - The reflected beam is **completely polarized** and emerges at $pprox 60^\circ$.
 - The transmitted beam emerges at $\approx 30^\circ$ but is only partially polarized.

Hence the correct statement is

(1) Reflected light is completely polarized and the angle of reflection is close to 60° .

Two identical charged conducting spheres A and B have their centres separated by a certain distance. Charge on each sphere is q and the force of repulsion between them is F. A third identical uncharged conducting sphere is brought in contact with sphere A first and then with B and finally removed from both. New force of repulsion between spheres A and B (Radii of A and B are negligible compared to the distance of separation so that for calculating force between them they can be considered as point charges) is best given as:



Solution:

Here is a detailed, step-by-step solution. We denote by r the distance between the centres of spheres A and B, and by k the Coulomb constant ($k = 1/(4\pi\varepsilon_0)$).

1. Initial force of repulsion

Before any contact, each of the two identical spheres A and B carries charge q. Since their radii are negligible compared to r, we treat them as point charges. Thus the initial force is

$$F=k\,rac{q\cdot q}{r^2}=k\,rac{q^2}{r^2}\,.$$

2. First contact: sphere C with A

Sphere *C* is initially uncharged (0), and *A* has charge *q*. On contact, two identical conductors share charge equally. So the total charge q + 0 = q is split equally:

$$q_A^{(\mathrm{new})} = q_C = rac{q}{2}\,.$$

3. Second contact: sphere C with B

After step 2, sphere C carries q/2, and sphere B still carries q. On contact, the total charge

 $q_B+q_C=q+rac{q}{2}$

is shared equally, so each ends up with

$$q_B^{(
m new)} = q_C^{(
m new)} = rac{1}{2} \, rac{3q}{2} = rac{3q}{4}$$

4. Final charges on A and BAfter removing C, the charges are

$$q_A=rac{q}{2}, \qquad q_B=rac{3q}{4}\,.$$

5. New force of repulsion

Again using Coulomb's law for point charges at separation r, the new force F' is

$$F' = k \, rac{q_A \, q_B}{r^2} = k \, rac{\left(rac{q}{2}
ight) \left(rac{3q}{4}
ight)}{r^2} = k \, rac{3q^2}{8 \, r^2} = rac{3}{8} \, k \, rac{q^2}{r^2} = rac{3}{8} \, F$$

```
Therefore, the new force of repulsion is \frac{3F}{8}, which is option (4).
```

29 Problem 29

A container has two chambers of volumes $V_1 = 2$ litres and $V_2 = 3$ litres separated by a partition made of a thermal insulator. The chambers contain $n_1 = 5$ and $n_2 = 4$ moles of ideal gas at pressures $p_1 = 1$ atm and $p_2 = 2$ atm, respectively. When the partition is removed, the mixture attains an equilibrium pressure of:

~



Solution:

Here is a complete step-by-step solution. We shall assume that when the (adiabatic) partition is removed the whole container remains thermally isolated, so that no heat is exchanged with the surroundings and no external work is done. Hence the total internal energy is conserved.

```
1. **Given data.**
Chamber 1:
moles n_1 = 5,
```

volume $V_1 = 2 L$, pressure $p_1 = 1$ atm. Chamber 2: *moles* $n_2 = 4$, volume $V_2 = 3 L$, pressure $p_2 = 2$ atm. Total after removal: *moles* $n = n_1 + n_2 = 9$, volume $V = V_1 + V_2 = 5 L$, common pressure p_f and temperature T_f to be found.

2. **Initial temperatures.**

From the ideal-gas law

Thus

 $T_i = rac{p_i V_i}{n_i R}, \qquad i = 1, 2.$ $T_1 = rac{p_1 V_1}{n_1 R} = rac{(1 ext{ atm})(2 ext{ L})}{5 R} = rac{2}{5} rac{ ext{atm} \cdot ext{ L}}{R} = 0.4 rac{ ext{atm} \cdot ext{ L}}{R},$ $T_2 = rac{p_2 V_2}{n_2 R} = rac{(2 ext{ atm})(3 ext{ L})}{4 R} = rac{6}{4} rac{ ext{atm} \cdot ext{ L}}{R} = 1.5 rac{ ext{atm} \cdot ext{ L}}{R}.$

3. **Energy conservation (adiabatic, no external work).** For an ideal gas $U = nC_vT$, so

$$U_1 + U_2 = U_f \quad \Longrightarrow \quad n_1 C_v T_1 + n_2 C_v T_2 = (n_1 + n_2) C_v T_f \,.$$

Since C_v is the same for both gases it cancels out, giving

$$n_1T_1 + n_2T_2 = (n_1 + n_2) \, T_f \quad \Longrightarrow \quad T_f = rac{n_1T_1 + n_2T_2}{n_1 + n_2} = rac{5\,(0.4) + 4\,(1.5)}{9} \; rac{\mathrm{atr}}{1}$$

4. **Final pressure.**

Again from the ideal-gas law for the final uniform state,

$$p_f = rac{n\,R\,T_f}{V} = rac{9\,R\,\left(rac{8}{9}\,rac{\mathrm{atmL}}{R}
ight)}{5\,\mathrm{L}} = rac{8\,\mathrm{atm}\cdot\mathrm{L}}{5\,\mathrm{L}} = 1.6\,\mathrm{atm}\,.$$

Answer: the equilibrium pressure is 1.6 atm (option 2).

30 Problem 30

A particle of mass m is moving around the origin with a constant force F pulling it towards the origin. If Bohr model is used to describe its motion, the radius r of the n^{th} orbit and the particle's speed v in the orbit depend on n as

(1)
$$r \propto n^{1/3}, \quad v \propto$$

(2)
$$r \propto n^{1/3}, \quad v \propto n^{2/3}$$

(3)
$$r \propto n^{2/3}, \quad v \propto n^{1/3}$$

(4)
$$r \propto n^{4/3}, \quad v \propto n^{-1/3}$$

Hide Solution

Solution:

Here is a detailed Bohr-model derivation.

Step 1: Centripetal-force condition

A particle of mass m moving in a circular orbit of radius r at speed v requires

$${mv^2\over r}~=~F$$

Step 2: Bohr quantization of angular momentum Bohr postulate (orbit n) gives

$$mvr = n\hbar \implies v = rac{n\hbar}{mr}.$$

Step 3: Eliminate vFrom the quantization,

Plug into the centripetal-force equation:

$$rac{m}{r} rac{n^2 \hbar^2}{m^2 r^2} = F \implies rac{n^2 \hbar^2}{m r^3} = F.$$

Hence

$$r^3 \;=\; rac{n^2 \hbar^2}{m \, F} \quad \Longrightarrow \quad r \;=\; \left(rac{\hbar^2}{m F}
ight)^{1/3} n^{2/3},$$

SO

 $r \propto n^{2/3}.$

Step 4: Speed dependence Using $v^2=Fr/m$ or directly $v=n\hbar/(mr)$, one finds

 $v~\propto~\sqrt{r}~\propto~n^{1/3}.$





- 2. Let
 - $T_M=687$ days be Mars' orbital period,
 - T_m be Mercury's orbital period (what we seek),
 - r_M and r_m their orbital radii.

We are told

$$r_M = 4 \, r_m$$
 .

 $T^2 \propto r^3$

3. Form the ratio of periods squared:

$$rac{T_M^2}{T_m^2} = rac{r_M^3}{r_m^3} = \left(rac{4 \, r_m}{r_m}
ight)^3 = 4^3 = 64 \, .$$

4. Hence

$$rac{T_M}{T_m}=\sqrt{64}=8 \implies T_m=rac{T_M}{8}=rac{687}{8}pprox 85.9 ext{ days}.$$

5. Rounding to the nearest whole day gives about 88 Earth days.

Answer: (1) 88 earth days.



experienced by the body due to the earth at a height equal to one-third the

radius of the earth from its surface is:
(1) 16 N
(2) 27 N
(3) 32 N
(4) 36 N
THide Solution
Solution: Here is a detailed solution:
1. Gravitational force (weight) at a distance r from the center of the Earth is $F(r) \;=\; {GMm\over r^2},$
where M is Earth's mass and m the body's mass.
2. On the surface ($r=R$) the weight is given as
$W_{ m surface} \;=\; rac{GMm}{R^2} \;=\; 48~{ m N}.$

3. At a height $h=rac{1}{3}\,R$ above the surface, the distance from the center is

$$r = R + h = R + rac{1}{3}R = rac{4}{3}R$$

Hence the new weight is

$$W_{
m height} \;=\; rac{G\,M\,m}{ig(rac{4}{3}Rig)^2} \;=\; W_{
m surface}\,rac{R^2}{ig(rac{4}{3}Rig)^2} \;=\; 48 \; imes\; rac{R^2}{rac{16}{9}R^2} \;=\; 48 \; imes\; rac{9}{16} \;=\; 27 \; {
m N}$$

4. Therefore the gravitational force at that height is **27 N**, which corresponds to choice (2).



Solution:
Here is a detailed solution.

Step 1: Resistance of each piece

The original wire has resistance R. Cutting it into 8 equal lengths makes each piece have resistance

$$r = \frac{R}{8}$$

Step 2: Four such pieces in parallel

Take four pieces, each of resistance r = R/8, and connect them in parallel. The equivalent resistance R_p of four identical resistors in parallel is given by

$$\frac{1}{R_p} = \frac{1}{r} + \frac{1}{r} + \frac{1}{r} + \frac{1}{r} = \frac{4}{r} \implies R_p = \frac{r}{4}.$$

Since $r = R/8$, we get
$$R_p = \frac{1}{4} \cdot \frac{R}{8} = \frac{R}{32}.$$

Step 3: Two such parallel-banks in series

We now have two identical blocks, each of resistance $R_p=R/32$, connected in series. The series combination simply adds:

$$R_{
m eq} = R_p + R_p = 2 \; rac{R}{32} = rac{R}{16}$$

Answer: $\frac{R}{16}$. This is choice (3).

34 Problem 34

Since

De-Broglie wavelength of an electron orbiting in the n = 2 state of hydrogen atom is close to (Given Bohr radius = 0.052 nm)

 $\overline{}$

(1) 0.067 nm
(2) 0.67 nm
(3) 1.67 nm
(4) 2.67 nm
THide Solution
Solution:
Here is a detailed step-by-step solution:
Step 1: De Broglie relation The de Broglie wavelength of a particle of momentum p is $\lambda \ = \ rac{h}{p} .$
Step 2: Bohr quantization of angular momentum In the Bohr model for the hydrogen atom the electron in the n th orbit satisfies
$mvr_n\ =\ n\hbar,$
so its momentum is $p \ = \ m v \ = \ rac{n \hbar}{r_n} .$
Step 3: Express λ in terms of the orbit radius r_n Substitute $p=n\hbar/r_n$ into the de Broglie relation:

$$\lambda \ = \ rac{h}{p} \ = \ rac{h}{n\hbar/r_n} \ = \ rac{h\,r_n}{n\hbar} \ = \ rac{2\pi\,r_n}{n} \,.$$

Step 4: Use the Bohr-radius formula The radius of the nth Bohr orbit is

$$r_n\ =\ n^2\,a_0,$$

where $a_0=0.052~\mathrm{nm}.$ Hence

$$\lambda_n \;=\; rac{2\pi\,r_n}{n}\;=\; rac{2\pi\,n^2a_0}{n}\;=\; 2\pi\,a_0\,n.$$

Step 5: Insert n = 2 and $a_0 = 0.052$ nm $\lambda_2 = 2\pi (0.052 \text{ nm}) \times 2 = 4\pi \times 0.052 \text{ nm} \approx 0.65$ nm ≈ 0.67 nm.

Therefore the closest choice is (2) 0.67 nm.

35 Problem 35

An electric dipole with dipole moment $5\times 10^{-6}\,\rm Cm$ is aligned with the direction of a uniform electric field of magnitude $4\times 10^5\,\rm N/C$. The dipole is then rotated through an angle of 60° with respect to the electric field. The change in the potential energy of the dipole is:

~

(1) 0.8 J

(2) 1.0 J

(3) I.∠ J

(4) 1.5 J

Hide Solution

Solution:

Here is a detailed, step-by-step solution. We denote the dipole moment by p, the field by E, and the angle between them by θ .

1. Potential energy of a dipole in a uniform field

$$U(\theta) = -\mathbf{p} \cdot \mathbf{E} = -p E \cos \theta$$

2. Initial configuration

The dipole is initially aligned with the field, so $heta_i=0^\circ$.

$$oldsymbol{U}_i=oldsymbol{U}(0^\circ)=-\,p\,E\,\cos0^\circ=-\,p\,E$$

3. Final configuration

The dipole is then rotated to $heta_f=60^\circ.$

$$U_f = U(60^\circ) = -\,p\,E\,\cos 60^\circ = -\,p\,E\,\, imes\,\,rac{1}{2} = -rac{1}{2}\,p\,E\,.$$

4. Change in potential energy

$$\Delta U = U_f - U_i = \left(-\frac{1}{2} pE\right) - \left(-pE\right) = \frac{1}{2} pE.$$

5. Substitute the numbers

$$p=5 imes 10^{-6}~{
m C}\cdot{
m m}, \quad E=4 imes 10^{5}~{
m N/C},$$

SO

$$pE = (5 imes 10^{-6})(4 imes 10^5) = 20 imes 10^{-1} = 2.0 \ {
m J},$$

and hence

$$\Delta U = rac{1}{2} \, pE = rac{1}{2} imes 2.0 \, \mathrm{J} = 1.0 \, \mathrm{J}.$$

Answer: The potential-energy increase is 1.0 J, so choice (2).

36 Problem 36

A constant voltage of 50 V is maintained between the points A and B of the circuit shown in the figure. The current through the branch CD of the circuit is:

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Hide Solution

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Solution:

Here is a systematic solution.

1. Circuit simplification

Because C and D are joined by a perfect conductor, they are at the same potential. Call that common intermediate node "M." Then the 1 Ω and 3 Ω resistors both go from A to M (in parallel), and the 2 Ω and 4 Ω both go from M to B (in parallel):



2. Find the equivalent resistances of the parallel sections

From A to M:

From M to B:

$$R_{A
ightarrow M} = rac{1 imes 3}{1 + 3} = rac{3}{4} = \ 0.75 \ \Omega.$$
 $R_{M
ightarrow B} = rac{2 imes 4}{2 + 4} = rac{8}{6} = \ 1.333 \ \Omega.$

3. Total resistance and total current In effect the circuit is now two series resistances $0.75~\Omega$ and $1.333~\Omega$, so

$$R_{
m eq} = 0.75 + 1.333 = 2.083 \ \Omega,$$

and with $V_{AB}=50\,\mathrm{V}$ the total current is

$$I_{
m tot} \;=\; rac{50}{2.083} \;pprox \; 24.0 \; {
m A}.$$

4. Node-M potential Taking $V_B=0$ and $V_A=+50$ V, the drop from A to M is

$$V_A - V_M = I_{
m tot}\,R_{A o M} = 24.0 imes 0.75 = 18~{
m V},$$

$$V_M = 50 - 18 = 32 \; {
m V}.$$

5. Individual branch currents

– Through 1 Ω (A \rightarrow M):

$$I_1 = rac{V_A - V_M}{1} = rac{50 - 32}{1} = 18 ext{ A}.$$

– Through 3 Ω (A \rightarrow M):

$$I_3 = rac{V_A - V_M}{3} = rac{18}{3} = 6 \ {
m A}.$$

– Through 2 Ω (M \rightarrow B):

$$V_2 = rac{V_M - V_B}{2} = rac{32}{2} = 16 \ {
m A}.$$

– Through 4 Ω (M \rightarrow B):

$$I_4 = rac{V_M - V_B}{4} = rac{32}{4} = 8 ext{ A}.$$

(Check: $I_1+I_3=18+6=24$ A into M, and $I_2+I_4=16+8=24$ A out of M.)

6. Current in the CD branch

At the top node C=M the current $I_1 = 18$ A arrives from the 1 Ω resistor, and $I_2 = 16$ A leaves through the 2 Ω . The remainder must flow down through the CD link. Thus

$$I_{CD} = I_1 - I_2 = 18 - 16 = 2 \text{ A},$$



where h is Planck's constant.

Step 2: Compute the photon wavelength. A photon of energy E has momentum

$$p_{
m ph}=rac{E}{c}\,,$$

so its de Broglie wavelength is

$$\lambda_{
m ph} = rac{h}{p_{
m ph}} = rac{h}{E/c} = rac{h\,c}{E}$$

Step 3: Compute the electron wavelength (non-relativistic). An electron of mass m and kinetic energy E satisfies

$$E=rac{p_{
m e}^2}{2m} \implies p_{
m e}=\sqrt{2mE}\,.$$

Hence

$$\lambda_{
m e} = rac{h}{p_{
m e}} = rac{h}{\sqrt{2mE}}$$

Step 4: Form the ratio $\lambda_{\rm ph}/\lambda_{\rm e}.$

$$rac{\lambda_{
m ph}}{\lambda_{
m e}} = rac{rac{h\,c}{E}}{rac{h}{\sqrt{2mE}}} = rac{h\,c}{E}\;rac{\sqrt{2mE}}{h} = c\;rac{\sqrt{2mE}}{E} = c\;\sqrt{rac{2m}{E}}\,.$$

Therefore the correct choice is



Hide Solution

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Solution:

Here is a full solution using uniform mass-density ρ and the parallel-axis theorem.

1. Masses of the two spheres

Let the big sphere have radius 2R and the small removed sphere have radius $R\!\!$. Then their masses are

$$M_{
m big} =
ho \; rac{4}{3} \pi (2R)^3 = rac{32}{3} \, \pi
ho \, R^3, \qquad M_{
m small} =
ho \; rac{4}{3} \pi R^3 = rac{4}{3} \, \pi
ho \, R^3.$$

After removal, the "rest" piece has mass

$$M_{
m rest} = M_{
m big} - M_{
m small} = rac{32-4}{3} \, \pi
ho \, R^3 = rac{28}{3} \, \pi
ho \, R^3.$$

2. Moment of inertia of the small sphere about the Y-axis

The small sphere is centered at (R, 0); the Y-axis is a distance d = R from its center. Its moment about its own center (any diameter) is

$$I_{ ext{cm,small}} = rac{2}{5} M_{ ext{small}} \, R^2,$$

and by the parallel-axis theorem

$$I_{
m small \ about \ Y} = I_{
m cm, small} + M_{
m small} \ d^2 = rac{2}{5} M_{
m small} R^2 + M_{
m small} R^2 = rac{7}{5} \ M_{
m small} \ R^2.$$

Substituting $M_{
m small}=rac{4}{3}\pi
ho\,R^3$ gives

$$I_{
m small} = rac{7}{5} \; rac{4}{3} \pi
ho \, R^5 = rac{28}{15} \, \pi
ho \, R^5.$$

3. Moment of inertia of the rest piece about the Y-axis First the big sphere about its central Y-axis:

$$I_{
m big} = rac{2}{5} \; M_{
m big} \; (2R)^2 = rac{2}{5} \; M_{
m big} \; 4R^2 = rac{8}{5} \; M_{
m big} \; R^2 = rac{8}{5} \; rac{32}{3} \pi
ho \, R^5 = rac{256}{15} \, \pi
ho \, R^5.$$

Removing the small-sphere part simply subtracts its moment:



40 Problem 40

A full wave rectifier circuit with diodes D_1 and D_2 is shown in the figure. If input supply voltage $V_{\rm in}=220\sin(100\pi t)$ volt, then at $t=15{
m msec}$



Here is a step-by-step analysis. We shall assume the secondary is centre-tapped so that each half–winding has half the total sinusoidal voltage.

1. Write down the input:

$$V_{in}(t)=220\sin(100\pi\,t)\quad {
m (volts)}$$

so the angular frequency is

$$\omega = 100 \pi ~{
m rad/s}\,, \quad f = rac{\omega}{2\pi} = 50 ~{
m Hz}, \quad T = rac{1}{f} = 20 ~{
m ms}.$$

2. Evaluate at t = 15 ms:

 $\omega t = 100\pi imes 0.015 = 1.5\pi ext{ rad } \implies V_{in}(0.015) = 220 \sin(1.5\pi) = 220 \cdot (-1) =$

3. In a centre-tapped secondary each half-winding has half the total amplitude, i.e.

 $V_{ ext{top-half}}(t) = 110 \sin(100\pi t), \qquad V_{ ext{bottom-half}}(t) = -110 \sin(100\pi t)$

both measured with respect to the centre tap (which is the return of the load).

4. At t = 15 ms we have

 $\sin(1.5\pi) = -1 \implies V_{ ext{top-half}} = -110 ext{ V}, \quad V_{ ext{bottom-half}} = +110 ext{ V}.$

5. Which diode conducts?

- The diode whose anode is at the more positive half-winding will be forward-biased. - Here the bottom half-winding is at +110 V (with respect to the centre tap), so the diode connected to the bottom half (that is D_2) is forward-biased.

– Conversely, D_1 is reverse-biased.

Answer:

 D_1 is reverse-biased and D_2 is forward-biased. (Option 2.)

41 Problem 41

Two gases A and B are filled at the same pressure in separate cylinders with movable pistons of radius r_A and r_B , respectively. On supplying an equal amount of heat to both the systems reversibly under constant pressure, the pistons of gas A and B are displaced by 16 cm and 9 cm, respectively. If the change in their internal energy is the same, then the ratio r_A/r_B is equal to

(1)
$$\frac{4}{3}$$

(2) $\frac{3}{4}$
(3) $\frac{2}{\sqrt{3}}$
(4) $\frac{\sqrt{3}}{2}$
(4) Hide Solution
Solution
Mere is the detailed solution:
Step 1. First law at constant pressure
For each gas (reversible, constant pressure)
 $Q = \Delta U + W$ with $W = P \Delta V$.
Step 2. Equal heat and equal ΔU^{}
We are given
 $Q_A = Q_B, \quad \Delta U_A = \Delta U_B.$

Hence from

$$Q_A = \Delta U_A + P \, \Delta V_A, \quad Q_B = \Delta U_B + P \, \Delta V_B$$

$$P\,\Delta V_A \;=\; Q_A - \Delta U_A \;=\; Q_B - \Delta U_B \;=\; P\,\Delta V_B \;\implies\; \Delta V_A = \Delta V_B.$$

Step 3. Relate ΔV to piston displacement Each piston has cross-sectional area $A=\pi r^2$, so

$$\Delta V_A \;=\; \pi \, r_A^2 \, h_A, \quad \Delta V_B \;=\; \pi \, r_B^2 \, h_B,$$

where $h_A=16\,\mathrm{cm}$ and $h_B=9\,\mathrm{cm}.\,$ Equating volumes,

$$\pi r_A^2 h_A \ = \ \pi r_B^2 h_B \quad \Longrightarrow \quad r_A^2 h_A = r_B^2 h_B \quad \Longrightarrow \quad rac{r_A^2}{r_B^2} = rac{h_B}{h_A} = rac{9}{16}$$

SO



Problem 42 42

A physical quantity, P is related to four observations a, b, c and d as follows: $P = a^3 b^2 / c \sqrt{d}$. The percentage errors of measurement in a, b, cand d are 1%, 3%, 2% and 4% respectively. The percentage error in the quantity P is

 \wedge

(2)	2%
(3)	13%
(4)	15%

Hide Solution

Solution:

Here is a step-by-step solution using the standard "log-differentiation" method for propagation of small percentage errors.

Step 1. Rewrite P in a form that makes exponents explicit:

$$P = \frac{a^3 b^2}{c \sqrt{d}} = a^3 b^2 c^{-1} d^{-1/2}$$

Step 2. Take the differential of $\ln P$. For small relative changes,

$$rac{{\mathrm d} P}{P} \,=\, 3\,rac{{\mathrm d} a}{a} \,+\, 2\,rac{{\mathrm d} b}{b} \,-\, 1\,rac{{\mathrm d} c}{c} \,-\, rac{1}{2}\,rac{{\mathrm d} d}{d}\,.$$

Step 3. Translate differentials into percentage errors. If $\delta a/a$ is the fractional error in a, then in percent it is $(\delta a/a) \times 100\%$. The problem gives:

$$rac{\delta a}{a} = 1\%\,, \quad rac{\delta b}{b} = 3\%\,, \quad rac{\delta c}{c} = 2\%\,, \quad rac{\delta d}{d} = 4\%\,.$$

Thus the fractional (percent) error in P is approximately

$$rac{\delta P}{P} \;=\; 3\! imes\!(1\%) \;+\; 2\! imes\!(3\%) \;+\; 1\! imes\!(2\%) \;+\; rac{1}{2}\! imes\!(4\%)$$

Step 4. Compute the sum:

3% + 6% + 2% + 2% = 13%.

Answer: The percentage error in P is 13%, i.e.\ option (3).



Hide Solution

Solution:

Here is a step-by-step solution using Malus's law. We have three polaroids in series:

1. P_1 at 0°,

- 2. P_m (the middle one) at 22.5° to P_1 ,
- 3. $\mathsf{P}_{_2}$ at 90° to $\mathsf{P}_{_1}$ (i.e.\ crossed with $\mathsf{P}_{_1}$).

Let the intensity just after P_1 be I_0 (this light is polarized along 0°).

Step 1: Transmission through the middle polaroid P_m

By Malus's law, when polarized light of intensity I_0 meets a polarizer whose axis makes an angle θ with the light's polarization, the transmitted intensity is

$$I_1 = I_0 \cos^2 \theta.$$

Here $heta=22.5^\circ$, so

$$I_1 ~=~ I_0 \cos^2(22.5^\circ).$$

Step 2: Transmission through the final polaroid P₂ The light emerging from P_m is polarized at 22.5°. P₂ is at 90°, so the angle between P_m and P₂ is

$$\phi = 90^{\circ} - 22.5^{\circ} = 67.5^{\circ}.$$

Again by Malus's law, the intensity after P_2 is

$$I_2 = I_1 \cos^2 \phi = I_0 \cos^2(22.5^\circ) \cos^2(67.5^\circ).$$

Step 3: Simplify using the identity $\cos(67.5^\circ) = \sin(22.5^\circ)$

$$I_2 = I_0 igl[\cos(22.5^\circ) igr]^2 igl[\sin(22.5^\circ) igr]^2 = I_0 igl[\sin(22.5^\circ) \cos(22.5^\circ) igr]^2.$$

But

$$\sin(45^{\circ}) = 2\sin(22.5^{\circ})\cos(22.5^{\circ}) \implies \sin(22.5^{\circ})\cos(22.5^{\circ}) = \frac{\sin(45^{\circ})}{2} = \frac{\sqrt{2}}{2}$$

Hence

$$I_2 = I_0 \Big(rac{\sqrt{2}}{4}\Big)^2 = I_0 \; rac{2}{16} = rac{I_0}{8}.$$



44 Problem 44

Two identical point masses P and Q, suspended from two separate massless springs of spring constants k_1 and k_2 respectively, oscillate vertically. If their maximum speeds are the same, the ratio (A_Q/A_P) of the amplitude A_Q of mass Q to the amplitude A_P of mass P is:

~



Hide Solution

Solution:

Here is a detailed step-by-step solution using valid $T_E X$ notation.

Step 1. Write down the expression for the maximum speed of a mass on a spring. For a simple harmonic oscillator of mass m, spring constant k, and amplitude A, the angular frequency is

$$\omega = \sqrt{rac{k}{m}}\,,$$

and the maximum speed is

$$v_{
m max} = A\,\omega = A\,\sqrt{rac{k}{m}}\,.$$

Step 2. Write v_{\max} for each mass. – For mass P (spring constant k_1 , amplitude A_P):

$$v_{P, ext{max}} = A_P \, \sqrt{rac{k_1}{m}}$$

– For mass Q (spring constant k_2 , amplitude A_Q):

$$v_{Q,\max} = A_Q \sqrt{rac{k_2}{m}}$$

Step 3. Use the condition "their maximum speeds are the same."

$$v_{P,\max} = v_{Q,\max} \implies A_P \sqrt{\frac{k_1}{m}} = A_Q \sqrt{\frac{k_2}{m}}$$

Step 4. Solve for the ratio A_Q/A_P . Divide both sides by $A_P\sqrt{k_2/m}$:

$$rac{A_P\,\sqrt{rac{k_1}{m}}}{\sqrt{rac{k_2}{m}}} \;=\; A_Q \;\; \Longrightarrow \;\; A_Q \;=\; A_P\,rac{\sqrt{k_1/m}}{\sqrt{k_2/m}} \;=\; A_P\,\sqrt{rac{k_1}{k_2}}\,.$$

Hence

$$rac{A_Q}{A_P} \;=\; \sqrt{rac{k_1}{k_2}}\,.$$



Solution:

Here is a detailed step-by-step solution.

Step 1. Fundamental frequency of the original pipe

A pipe of length L open at both ends has its fundamental (first-harmonic) frequency given by

$$f = rac{v}{2L},$$

where \boldsymbol{v} is the speed of sound in air.

Step 2. New boundary conditions when half-submerged When the pipe is dipped vertically into water up to half its length, the water surface

inside the pipe acts as a **rigid (closed) end** for the air column. Thus:

– The top end of the air column is still open \rightarrow displacement antinode (pressure node).

– The water surface is effectively a closed end \rightarrow displacement node (pressure antinode).

But now the *length of the air column* above the water is only

Step 3. Fundamental frequency of an open–closed pipe An open–closed pipe of length L' has its fundamental (first-harmonic) frequency

$$f' \;=\; rac{v}{4\,L'}.$$

Substituting $L' = \frac{L}{2}$, we get

$$f' \;=\; rac{v}{4\,(L/2)} \;=\; rac{v}{2L} \;=\; f.$$

The new fund	mental frequency is the same as before, namely	
	$\fbox{f'}=f.$	
Hence the co	ect choice is **(2) f .**	

